Sensitivity analysis and optimization of solutions computed through the ANM

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The Asymptotic Numerical Method (ANM) allows for the solution (in u and λ) of sufficiently smooth nonlinear PDE problems $R(u(a), \lambda(a)) = 0$ using truncated Taylor expansions [3] expanded with respect to the path parameter a. This yields a sequence of linear systems

$$\left\{R_{1|u_1=id,\lambda_1=0}\right\} u_k + \left\{R_{1|u_1=0,\lambda_1=1}\right\} \lambda_k = -\left\{R_{k|u_k=0,\lambda_k=0}\right\}, \tag{1}$$

where u_k and λ_k are Taylor coefficients of u and λ at order k. Among others, these systems depend on the same tangent linear matrix $\{R_{1|u_1=id,\lambda_1=0}\}$ and some higher order differentiation terms $\{R_{k|u_k=0,\lambda_k=0}\}$ that are Taylor coefficients of R computed with peculiar initialisations. Differentiations were hand-coded for a long time. We recently propose an AD implementation, named Diamant [2], based on operator overloading.

In the first part of the talk, we are interested in sensitivity computations of solutions computed through the ANM. Let solutions u and λ depend on a mechanical parameter p. In the ANM context, a sensitivity computation thus involves mixed derivatives of u and λ with respect to a and p. This yields

$$\begin{aligned} &\left\{R_{1|u_1=id,\lambda_1=0}\right\} u_k^d + \left\{R_{1|u_1=0,\lambda_1=1}\right\} \lambda_k^d = \\ &= -\left\{R^d_{1|u_1=id,\lambda_1=0}\right\} u_k - \left\{R^d_{1|u_1=0,\lambda_1=1}\right\} \lambda_k - \left\{R^d_{1|u_1=0,\lambda_1=0}\right\}, \end{aligned} \tag{2}$$

where R^d_k,u^d_k, λ^d_k are derivatives of Taylor coefficients R_k, u_k, λ_k with respect to p. These mixed derivatives may be (i) obtained in a semi-automatic fashion applying an AD tool to an old hand-coded ANM computer program, or (ii) fully automated through a combination of AD tools (Diamant and Adimat[1]). Both methods are discussed on an academical PDE problem implemented in Matlab. The tangent linear differentiation is performed using Adimat.

In the second part of the talk we present some optimisation results. The nonlinear free vibration of a sandwich plate (steel/polymer/steel)[4] is implemented in a Fortran code, the viscoelastic core being modeled by the constitutive law $E(w) = E^R(w) + iE^I(w)$. This code is differentiated using Tapenade. The goal of the optimisation is to identify frequency dependent parameters $E^R(w)$ and $E^I(w)$ from experimental data (resonant frequencies and modal loss factors) provided by the industiral company.

References

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